STRUCTURAL AND ALGORITHMIC METHODS INCREASING RELIABILITY OF THE MEASUREMENT INSPECTION

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Abstract: The auxiliary quantity, homogeneous with the measured one, is proposed to apply in testing of the object, which is measured throw the measurement channel. The result of processing this quantity in measurement channel is used for the additive and multiplicative adjustments of settings limits for the positive decision. This allows to reduce the impact of imperfect processing characteristics in the measurement control. The valuable effectiveness of the correction for different relations of the processing error components and the influence of different relations of the auxiliary quantity components are shown. The influence of errors of forming auxiliary quantity is also estimated.

Keywords: measurement channel, additive and multiplicative adjustments, settings limits, the measurement control, auxiliary quantity, equal intervals.

1. Introduction

The reliability of measurement inspection is a confidence measure in testing used for decisionmaking. This inspection is characterized by its uncertainty [1, 2]. If the inspection is based on a measurement then quantity X is measured and it contains information about the specific properties of a controlled object [3]. The comparison procedure of the value x with limits x_l, x_h of the tolerance interval is preceded by the sequence of "measurement/processing" operations of this quantity in a measuring channel (MC). The real characteristics $\varphi(x)$ of measuring channel differs from its nominal characteristics $\varphi_0(x) = x$. The decision of the inspection can be taken only after processing x by the channel. The decision is described by inequality $\varphi_0(x_l)<\varphi(x)<\varphi(x_h)$, which does not coincide with the required decisionmaking rule $x_l \leq x \leq x_h$. If the channel output quantity is calibrated in units of the measured quantity then $\varphi_0(x) = x$ and $\varphi(x) \approx (x+\Delta)(1+\gamma)$ will be obtained. The function $\varphi(x)$ can be expressed as a series expansion

$$\varphi(x) - \varphi_0(x) = a + (b-1)x + \sum_{i=2}^{n} c_i x^i \; . \qquad (1)$$

The main systematic error, resulting from the nonideal (real) MC characteristics, commonly contains in practice two components: additive $\Delta= a/b$ and multiplicative coefficient $\gamma=(b1)$, because a third term describing the nonlinearity of MC characteristic is usually negligible. Combinations of these components, both as absolute values and signs, result in the correctness of inspection to a different extent. This effect was analyzed in [4]. There were shown that the effect of nonideal characteristics of the measurement channel might be included with the use of additional segments θ_l and θ_h. Their lengths are dependent on the combination of two error components. These segments cause the shift of x interval. If a decisionmaking rule is transformed to $x_l + \theta_l \leq x \leq x_h + \theta_h$ then probability of the incorrect decision is defined by area under the curve of the probability distribution of controlled quantity. Type of wrong decisions, false refuse or unfounded refuse, depends on the position of segments θ_l and θ_h in relation to the lower x_l and upper x_h limits of the tolerance interval.

The traditional way to improve the reliability of measurement inspection involves the use of more accurate measuring instruments in the channel MC. Typically, this reduces the speed and increases the costs of control. Also not always, there is a suitable measuring equipment available. An influence of deviation, between the real processing characteristics and the nominal one, on the correctness of inspection can be reduced by other means. One of the methods is described below. The purpose of this paper is to present the ways to increase reliability of decisions taken in an inspection process and to reduce incorrect decisions [5]. It is proposed to achieve this without employment of more precise measuring equipment. This will be done by reducing the impact of errors in measuring channel MC.

2. Theoretical background of the method

In the inspection process, in contrast to the measurements, one cannot determine the precise value of a measured physical quantity x of controlled object. It is sufficient to determine whether it is within the tolerance interval $x_l \leq x \leq x_h$. Therefore, the obtained result does not need to be corrected for reducing the resultant error of sequence operation "measurement /processing" in the measuring channel. Instead of that, a number of other equations, obtained in a structural or algorithmic way which contain information about the real characteristics $\varphi(x)$ of the measuring channel, is possible to use. The calculations based on the results obtained from these equations are used for adjusting the limits of the acceptance interval of x values. This adjustment is made to bring the relationship between result of processing $\varphi(x)$ and adjusted settings $\varphi_0(x_l)$, $\varphi_0(x_h)$ into conformity with the relationship between x and predefined limit values x_l and x_h of the tolerance interval. If the correction associated with calculating the correction d, which is added to settings, then it is additive correction. And if settings are multiplied by the multiplier c, then it is the multiplicative correction.

An auxiliary quantity x_0, homogeneous with the measured one, should be applied to implement the method for increasing the reliability of measurement inspection. If the value x_0 is known then it will be also known the result of its ideal processing $\varphi_0(x_0)$. Divergences between real $\varphi(x_0)$ and ideal result $\varphi_0(x_0)$ are considered in calculations of correction a, or in adjustment of b parameter. Thus, before the actual control procedure, auxiliary quantity x0 should be given to input of the MC measuring channel. The result of its processing $\varphi(x_0)$ contains information about the actual channel characteristics (Fig. 1).

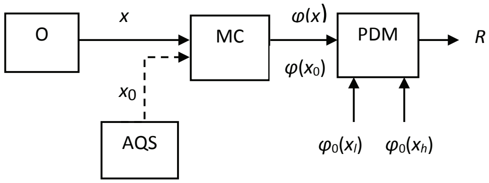


Fig. 1. Method for increasing the reliability of measurement inspection: O – controlled object; AQS auxiliary quantity x0 source; MC – measuring channel; PDM – processing and decision making module, R result of inspection

Additive correction. As the result $\varphi_0(x_0)$ of ideal processing of x_0 is known, on the basis of $\varphi(x_0)$ and $\varphi_0(x_0)$ one can determine the proportional factor $d{=}\varphi(x_0)\ \varphi_0(x_0)$ of the multiplicative component of measuring channel error. Value d is used for additive correction i.e. to shift the initial settings

$$\varphi_a(x_l) = \varphi_0(x_l) + d\,,\ \varphi_a(x_h) = \varphi_0(x_h) + d\ .$$

In this way the additive correction was done. Without loss of generality it can be assumed that the characteristics, real and nominal, are as follows

$$\varphi(x) = (x + \Delta)(1 + \gamma),\ \varphi_0(x_0) = x,$$

where: Δ – additive component and γ – multiplicative component of absolute processing error.

Decision rule "Update" for the input of measurement channel is as follows

$$\frac{x_l}{1+\gamma} - \Delta < x < \frac{x_h}{1+\gamma} - \Delta\ .$$

After adjusting the settings for value $d = x_0\gamma + \Delta(1+\gamma)$ one receives

$$\frac{x_l}{1+\gamma} + x_0\frac{\gamma}{1+\gamma} < x < \frac{x_h}{1+\gamma} + x_0\frac{\gamma}{1+\gamma}\ . \qquad (1)$$

It results from (1) that the additive error does not affect the test results. To separate the ideal decision, the rule expression (1) is converted to the following

$$x_l + \alpha(x_0 - x_l) < x < x_h + \alpha(x_0 - x_h), \qquad (2)$$

$\alpha \equiv \frac{\gamma}{1+\gamma}$ where .

The right parts from both sides of the equation (2) include, remaining after correction, partial impact of measuring channel errors. Substituting different values x_0 in the expression (2), the tolerance interval can be changed. From the expression (2) it results that probability and type of incorrect decisions depend on the values x_0, x_l, x_h. There are three cases of selecting the value x_0: within the tolerance interval, and two out of its limits, i.e. $x_0 < x_l$, $x_l < x < x_h$, $x_0 > x_h$.

From the detailed analysis in [4] it results that the impact of measuring channel error on the reliability of measurement inspection will be minimal when x_0 will be selected within the tolerance interval. Then the decision rule after correction of limits will be

$$x_l + \alpha(x_0 - x_l) < x < x_h - \alpha(x_h - x_0) \qquad (3)$$

or

$$x_l + \theta_l \leq x \leq x_h - \theta_h\ . \qquad (4)$$

Equal intervals θ_l and θ_h capture the impact of measuring channel errors on accuracy of inspection. The area under the probability density function (pdf) for the value of controlled quantity is proportional to the probability of incorrect decisions (Fig. 2).

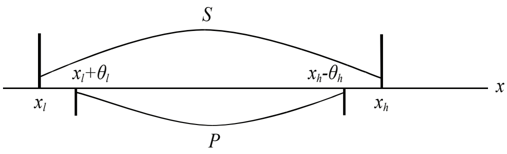
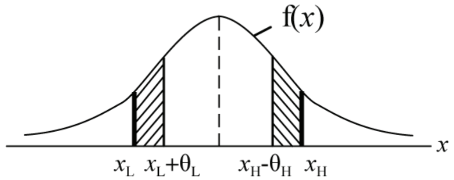


Fig. 2. Location of the intervals of parameter x for the objects which met efficiency standards S (tolerance interval) and found as fit F (acceptance interval)

The length of intervals θ_l and θ_h has the positive sign. A sign in (4) gives information about a type of incorrect decision. Thus, in the case of controlled objects whose value of information parameter is within the range θ_l and θ_h , it can only take place a *false refuse* due to their uselessness although in fact they are efficient (Fig. 3). The probability of a false refuse determines the expression

$$P_{err} = P_f = f(x_l)\,(\theta_l + \theta_h) + \tfrac{1}{2} f(x_l)\,(\theta_l^2 + \theta_h^2) \quad (5)$$

where: $f(x_l)$ – density distribution of the possible values of controlled quantity for x_l.



For uniform distribution the second component on the right side of (5) disappears. If the multiplicative component of measuring channel error is negative, there is only a decision: *unfounded refuse.*

Let us consider the impact of error x_0 on the effectiveness of additive correction setting limits decision. In this case the value $\tilde{x}_0 = x_0 + \Delta_0$ is given to the input of measuring channel with an error Δ_0. The result of ideal processing of this quantity will not be equal to $\varphi_0(x_0)$. Then, according to the algorithm $\tilde{b}$ is calculated instead of *b*. After the additive correction one receives

$$x_l + \alpha(x_0 + x_l) - \Delta_0 < x < x_h - \alpha(x_h - x_0) - \Delta_0 . \quad (6)$$

It results from equation (6) that on the one hand an error of auxiliary quantity increases the replacement interval – see right side of the equation and on the other hand it reduces the interval – see on the left side of the equation. The only one requirement is that the value $\tilde{x}_0$ should be within the initial tolerance interval. To fulfill this condition, assuming an uniform distribution of controlled size, the auxiliary quantity error will not affect the conformity of inspection. The optimal selection of auxiliary quantity value is $(x_h+x_l)/2$ assuming the normal distribution of a controlled variable value and the positive multiplicative component of measuring channel error. However, for the negative multiplicative component of error, a reduction of the impact of remnant errors (remained after correction) occurs for $x_0=x_l$ or $x_0=x_h$. In setting a final probability of incorrect decisions there is need to consider the second component of right side of equation (3). It depends on the ratio of the standard deviation of the dispersion values in controlled amounts σ and the interval duration $(x_h x_l)$ of limit values of controlled variable x. In this case, as opposed to a dependence for uniform distribution, the auxiliary quantity error increases the probability of incorrect decision. However, an additional increase in the probability of incorrect decisions will be of the second order of smallness.

Multiplicative correction. As it is apparent from the expressions (3) and (6), the probability of incorrect decisions depends on the multiplicative component of MC error. If preliminary analysis shows that the effect is significant, there is a need to perform multiplicative correction. With this correction the factor c of both real and ideal results of auxiliary quantity processing x_0 is equal to $c = \dfrac{\varphi(x_0)}{\varphi_0(x_0)}$. This factor is used to correct the limits of tolerance interval. If the real function of the channel is $\varphi(x) = (x + \Delta)(1 + \gamma)$, then $c = \dfrac{(x_0 + \Delta)(1 + \gamma)}{x_0}$. After correction a decision rule is as follows

$$\frac{x_h}{x_0}(x_0 + \Delta) < (x + \Delta) < \frac{x_h}{x_0}(x_0 + \Delta).$$

From the last expression it results that the multiplicative error component does not affect the test result. Similarly, as in the previously discussed case, one stands out ideal (nominal) decision rule $x_l < x < x_h$. Then one goes to inequality in which additional restrictions, on the right side and the left side, take into account the impact of nonideal processing characteristics of the MC

$$x_l + \frac{\Delta}{x_0}(x_l - x_0) < x + \Delta < x_h + \frac{\Delta}{x_0}(x_h - x_0). \quad (7)$$

Thus, the influence of multiplicative component of the measurement error on the reliability of measurement inspection was eliminated.

To minimize the remnant impact of constant value a of error, the auxiliary value x_0 within the permissible range should be also chosen. However, using a multiplicative correction, regardless of the probability distribution of a controlled quantity value x, the selected value x_0 should be near to x_h. Decision rule determines the reliability of decisions:

$$x_l - \frac{\Delta}{x_0}(x_0 - x_h) < x < x_h + \frac{\Delta}{x_0}(x_h - x_0). \quad (8)$$

From a comparison of expressions (3) and (8) it results that a type of incorrect decisions changes to opposite then for the additive correction (in this case a failure will be detected).

Let us also consider the impact of error of creating the auxiliary quantity x_0 on the effectiveness of the multiplicative correction. As a result of inaccuracy during the creation of this quantity, i.e. when $\widetilde{x}_0 = x_0 + \Delta_0$ then the real value of the correction factor is

$$\widetilde{c} = \frac{(x_0 + \Delta + \Delta_0)(1+\gamma)}{x_0}. \quad (9)$$

If the decision rule is selected taking into account the error of creation x_0 then

$$x_l - \frac{\Delta}{x_h}(x_h - x_l) + \frac{\Delta_0}{x_h} x_l < x < x_h + \Delta_0 ,$$

$$\frac{\Delta_0}{x_h} < 1. \quad (10)$$

From the expression (10), the effect of errors during creation the quantity x0 can be expressed as a relatively small reduction in the length of the smaller equivalent interval, and equivalent increase the length of the upper replacement interval, proportional to the error of forming the auxiliary quantity. This may increase the probability of incorrect decisions and significantly reduces the effectiveness of the multiplicative correction.

3. Numerical example

The functionality of a product is characterized by a certain output quantity x. According to the requirements of production technology this quantity can occur with equal probability in the range from 0 to 20 mV. These objects should be selected whose x value is within the interval (10 14) mV, thus $\varphi_0(x_l) = 10$, and $\varphi_0(x_h) = 14$. The real characteristic of the measuring channel is

$$\varphi(x) = (x + 0.1)(1 + 0.1).$$

According to the equation (6) the probability of incorrect decisions with uniform probability density distribution of controlled quantity $f(x) = 0.05$, will be $P_{err} = 0.13$, i.e. the conformity of inspection is 0.87. Auxiliary quantity x0 was used, located within the tolerance interval (1014) mV. After appropriate corrections, the probability of incorrect decisions decreased to $P_{err} = 0.02$. So the conformity of inspection increased to 0.98. For $x_0 = 12$ mV permissible error of auxiliary quantity cannot exceed ±*2* mV (±*17*%). Error of auxiliary quantity creation does not affect the conformity of this inspection.

When choosing $x_0 = 14$ mV and an additive error $\Delta = 0.1$ mV, with a multiplicative correction, the probability of incorrect decisions will be $P_{err} = 0.002$. Thus, the probability is decreased by an order of magnitude. The impact of error in creating the auxiliary quantity x_0 will be also considered. When $\Delta_0 = \pm 2$ mV, an additional component of probability of incorrect decisions is *0.025*. This corresponds to the reliability of inspection $D = 0.975$ – by an order of magnitude in relation to the additive correction of settings. In Table 1 the reliability of inspection is shown. It is calculated for the case where the processing characteristics of the measuring channel MC has the form of $\varphi(x) = (x + 0.1)(1 + 0.1)$.

From Table 1 it results that by reducing the impact of measuring channel error, the correction allows to increase the reliability of inspection. In the presented example

$x_i = 10$ mV, $\alpha = \frac{\gamma}{1+\gamma} = 0.999$ and Δ=0.1 mV,

i.e. $\alpha x_l > \Delta$.

Therefore, more appropriate is the use of a multiplicative correction. The data in Table 1 confirm that the reliability of inspection will be greater then. It also results that the reliability of the multiplicative correction decreases as the auxiliary quantity x_0 is formed with an error. This should be taken into account when choosing the type of correction. A type of incorrect decision is also significant.

4. Conclusions

Appropriate correction of the tolerance interval of measured quantity reduces the impact of the error components of the measurement channel on the reliability of inspection. An impact of additive error component is completely eliminated after applying the additive correction. At the same time accuracy requirements decreases concerning the auxiliary quantity. However, if the multiplicative component of processing error in measuring channel is dominant, the effectiveness of additive correction decreases.

Multiplicative correction of the tolerance interval of measured quantity eliminates the influence of multiplicative error component. At the same time the auxiliary quantity error causes a proportional increase in the probability of incorrect decisions. Thus, the choice of correction depends on the ratio of both components: the measuring channel error and the accuracy in creating the auxiliary quantity. Taking into account the probability distribution of controlled quantity and properly selecting the value of auxiliary quantity, the impact of a multiplicative component of error in measuring channel can be minimized. Depending on the sign of this component, the results of inspection may include falsely detected or undetected malfunctions.

Table 1. The reliability of inspection and the types of incorrect decisions for different correction of limits

Way of limits correction	Reliability of inspection		Types of incorrect decisions
	Error of x_0		
	negligible	10%	
Without correction	0.87	0.87	for x_l – unfounded refuse for x_h – false refuse
Additive correction	0.98	0.98	for x_l and x_h – false refuse
Multiplicativ e correction	0.998	0.895	for x_l and x_h – unfounded refuse

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СТРУКТУРНОАЛГОРИТМИЧЕСКИЕ МЕТОДЫ ПОВЫШЕНИЯ ДОСТОВЕРНОСТИ ИЗМЕРИТЕЛЬНОГО КОНТРОЛЯ

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Резюме: Показано, что для уменьшение влияния погрешности измерения на достоверность контроля используется вспомогательная величина, однородная с измеряемой. Результат преобразования этой величины используется для аддитивной и мультипликативной коррекции уставок. Это позволяет уменьшить влияние неидеальности характеристики преобразования при измерительном контроле. Показана эффективность коррекции при различных соотношениях составляющих погрешности преобразования. Оценено влияние погрешности формирования вспомогательной величины. Рассмотрен числовой пример.

Ключевые слова: измерительный канал, аддитивная и мультипликативная коррекция, уставки, измерительный контроль, вспомогательная величина, эквивалентные интервалы.

СТРУКТУРНИ И АЛГОРИТМИЧНИ МЕТОДИ ЗА ПОВИШАВАНЕ НА ДОСТОВЕРНОСТТА НА ИЗМЕРВАТЕЛНИЯ КОНТРОЛ

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Резюме: В доклада е показано, че за намаляване на влиянието на грешката от измерването на достоверността на контрола се използва помощна величина, еднородна с измерваната. Резултатът от преобразуването на тази величина се използва за адитивна и мултипликативна корекции на границите на настройките. Това позволява да се намали влиянието на неидеалността на характеристиката на преобразуване при измервателния контрол. Посочена е ефективността на корекцията за различни съотношения на съставящите компоненти на грешката на преобразуване. Оценено е влиянието на грешката от формирането на помощната величина. Разгледан е числов пример.

Ключови думи: измервателен канал, адитивна и мултипликативна корекция, граници на настройки, измервателен контрол, помощна величина, еквивалентни интервали.