1. Introduction
The development of various areas of science and technology associated with the use of radio signals is characterized by the development of ever higher frequencies of electromagnetic oscillations. The last decades were marked by rapid development of the microwave range. At present, microwave radio signals are widely used in the communications, radiometry, radiolocation, medicine and household appliances, charged-particle accelerators, and in testing of microwave devices, mobile phones and other wireless devices that are usually built on the basis of microwave signals and tract theory. At microwaves, instead of the usual currents and voltages, the power and the complex coefficient of termination reflection are used to fully characterize the generator and load parameters. The complex nature of the reflection of the termination means that it can be fully described by the modulus and phase of the reflection coefficient. The object of measurements at microwave frequencies is a standing wave, which is set in the tract between the generator and the termination. In such measuring devices as vector network analyzers for obtaining information on both the module and the phase of the reflection coefficient, the heterodyne transformation method is used, which leads to complexity and high cost of vector network analyzers. However, more economic and at the same time no less precise scalar network analyzers were proposed by G.F. Engen [1]. The peculiarity of scalar network analyzers is that, its power sensors measure only the amplitude of the standing wave in a tract without the phase. But then, as a result of the calculations, information on both phase and amplitude is extracted from the sensor signal readings. These scalar network analyzers are also called six-port, because of their construction: the first input port is port where the generator is connected, the output port is one where the termination is connected, the other four ports are for connecting the power sensors (Fig. 1). The six-port analyzer or reflectometer, as it measures reflection coefficient of the termination, can be classified by microwave block construction as the six-port reflectometers that are quite different in design of the microwave unit. But in proposed version of the six-port reflectometer – multiprobe microwave multimeter [2, 3], the microwave unit is the simplest, such that the sensors are placed along a standing wave in the tract and along the tract itself at certain distances. The problem of the given research can be presented as a contradiction between the fact that measurements are made by multimeter sensors readings only, and the remaining parameters are obtained from calculations based on algorithms, therefore, the adequacy and accuracy of the models are very important. The termination of the multimeter, i.e. the open end of a coaxial line closely contacts with the dielectric material, so dielectric permittivity of the dielectric can be measured [3]. Being a non-destructive method, this method has advantages over other methods of measuring dielectric permittivity, since it does not require special preparation of the sample, it is characterized by a simple experimental procedure and attracts the attention of specialists studying the properties of biological objects and tissues of living organisms on domain of microwave frequencies. The possibility of simultaneous measuring the dielectric permittivity, along with measurements of the incident, reflected, transmitted power, the modulus and phase of the reflection coefficient and the wavelength, makes the multiprobe microwave multimeter even more versatile. The determination of dielectric permittivity is made on the basis of

Annotation. Multiprobe microwave multimeters allow to measure dielectric permittivity of materials along with power, complex reflection coefficient and wavelength, being universal devices. A method is proposed for determining the permittivity of materials on the basis of the model in the form of an integral equation that is distinguished by the use of the dyadic Green's function as the kernel of the integral equation, with the simplification by variational principles are used to obtain engineering formulas for calculating measurement uncertainty.

Keywords: dielectric permittivity, complex reflection coefficient, multiprobe microwave multimeter, model, sensor signal, processing algorithm, measurement uncertainty
information about the modulus and the phase of the complex reflection coefficient, and it leads to increasing of the uncertainty of the complex reflection, as to the uncertainty of indirect measurements. First, consider the multimeter and ways and means of improving its accuracy, and then – how with the aid of a microwave multimeter one can accurately measure the dielectric permittivity.

2. Multiprobe microwave multimeter algorithm

A multiprobe microwave multimeter is a part of a transmission line with sensors whose signals are processed by computing devices using algorithms, for example, in the waveguide frequency range phase distance is

\[
\cos \theta = \frac{P_1 - P_4 - (P_2 - P_3)}{2(P_2 - P_3)}, \tag{1}
\]

Reflection coefficient phase

\[
\varphi = \arctg \left( \frac{(P_1 - P_3)2(\cos \theta - 1)}{\sin \theta(P_1 + P_3 - 2P_2)} \right), \tag{2}
\]

reflection coefficient modulus

\[
\Gamma = -\cos(\varphi + \theta_1) + \sqrt{\cos(\varphi + \theta_1)^2 - \left(1 - \frac{P_1}{P_{inc}}\right)}, \tag{3}
\]

average power

\[
P = \frac{0.5(P_1 + P_3) - P_2 \cos \theta}{1 - \cos \theta}, \tag{4}
\]

passing power

\[
P_{pass} = \sqrt{\frac{P_2}{4\sin^2 \theta} (P_1 + P_3(1 + \cos \theta) - (P_1 - P_3)^2)} \tag{5}
\]

incident power

\[
P_{inc} = \frac{P + P_{pass}}{2}, \tag{6}
\]

wavelength

\[
\lambda = \frac{2\pi L}{\arccos \left( \frac{P_1 - P_4 - (P_2 - P_3)}{2(P_2 - P_3)} \right)} \tag{7}
\]

where \(P_{inc}\) is the incident power, \(P_{pass}\) – passing power, \(\Gamma\) – is the reflection coefficient modulus, \(\varphi\) – is the reflection coefficient phase, \(\lambda\) – I the wavelength, \(\theta = \frac{2\pi L}{\lambda}\) – is the phase distance between the sensors, \(L\) – is the physical distance between the sensors, \(P\) – is the average power, \(P_i\) – is the sensor signal, \(i\) – is the sensor number.

The further increasing in the number of sensors in a multi-probe microwave multimeter with a microwave unit in the form of the waveguide and the sensors mounted therein is easy to realize. And the benefit of this approach is to reduce a type A uncertainty by processing and averaging. The averaging can be applied from simple methods to rather complex ones, for example, the method of least squares (OLS) [4], but a problem arises in the form of the effect of the sensors on each other. However, this is a type B uncertainty that can be estimated and a correction factor is applied to results of measurement and calculation. With the help of the theory of microwave circuits and flow chart graphs, such calculations were made, and the use of increasing the number of sensors is shown [5, 6]. The describing of the reflections between sensors with the theory of microwave circuits has an alternative. It is full-field approach with the use of the dyadic Green’s functions, which makes it possible to further improve the accuracy due to the account on higher order modes of the waveguide [7]. It remains to be noted about the conditions of measurement. The microwave multimeter is used for measurements in “hot mode”, that is, without turning off the generator. The multimeter is located between the generator and the termination. The power level of the generator is determined by the technical specification. With the same multiprobe method and operation algorithms, the multimeter can be used both for heating and drying or in antennas at a high power level, and for non-destructive testing at low power levels with a suitable generator. The generator is usually not a part of the multimeter. Also, the knowledge of dynamic range is necessary to select the type of multimeter sensors. For example, high level power sensors are thermocouples, low power level sensors are bolometers and diodes.
3. Dielectric permittivity measurement using multiprobe microwave multimeter

Let the termination of the microwave multiprobe multimeter represents the open end of a coaxial line loaded on a dielectric with an unknown dielectric permittivity. The measuring circuit is shown in Fig. 1. A microwave oscillator 1 is connected to a multiprobe power converter 2. The converter output is connected to a probe antenna 4 whose open-ended section is loaded by a sample of the investigated dielectric 5. The readings of the multiple-probe converter are processed by the readout device 3 that includes a personal computer. A microwave multimeter acts as the converter 2 and readout device 3 [1].

Fig. 1 Scheme of measurement using a microwave multimeter.

The formulas are known for the engineering calculations of dielectric permittivity [8]

\[ Y = G + jB, \]  
\[ G = \frac{1 - \Gamma^2}{1 + \Gamma^2 + 2\Gamma \cos \phi}, \quad B = \frac{-2 \Gamma \sin \phi}{1 + \Gamma^2 + 2\Gamma \cos \phi}, \]  
\[ Y = \frac{j \kappa \varepsilon Y_0}{\ln(b/a)} \int_0^\infty \left[ J_0(\xi a) - J_0(\xi b) \right]^2 \frac{d\xi}{\xi \sqrt{\xi^2 - k^2 \varepsilon}}, \]  

where \( Y_0 \) – is the characteristic impedance of the coaxial line, \( a \) – is the internal dimensions of the coaxial section, \( b \) – is the outer dimensions of the coaxial section, \( \varepsilon = \varepsilon' - j\varepsilon'' \) – is the complex dielectric permittivity of material, \( \varepsilon' \) – is the permittivity of the coaxial line’s filling material, \( \xi \) – is the integral variable, function \( J_0 \) is a first-type Bessel function of zero order.

The task of the study was to modify and refine expression (10), in order to improve accuracy.

And for metrological purpose, numerical methods or analytical cumbersome expressions are highly undesirable, because after modification, an expression suitable for determining the error of indirect measurement has to be obtained. The critical study of literature sources and comparison with known methods showed the origin of the relation (10) from three components, namely, the integral equation, variational principles, and the scalar Green function. Note that the scalar Green function is good for antennas application and is used for the far field. A similar problem was solved by the Rayleigh-Ritz method. The Rayleigh-Ritz method became the basis of the finite element method (FEM) [9] and the finite element method is implemented in the form of many modern computer mathematical application. But for metrological purposes in the classic form, the FEM is not very suitable. The material dielectric permittivity measurement model with the near field open end of a coaxial sensor is now studied [10] and it has an analytic appearance. That is, in proposed model an integral equation is used, the kernel of which is the dyadic Green’s function.

4. Conclusion

A method for determining the permittivity of materials based on the integral equation and variational principles is proposed, the latter is used to simplify the integral equation by taking into account only the leading term, which is different from the known approach by the type of the kernel of the integral equation. Now it is a dyadic Green’s function.

5. References


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