RESEARCH OF INFLUENCE OF MAINS FREQUENCY DEVIATION ON VOLTAGE SPECTRUM MEASUREMENT ERROR BY DFT METHOD

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Abstract: Measurements of many power quality indicators are based on a spectrum measurement by means of DPF as the effective algorithms of its computation having the general name FFT are developed. However in case of the fixed sampling frequency and number of counting the mains frequency deviation from rated value causes the distortions of spectrum called by spectrum leakage. To reduce spectrum leakage, sampling frequency is set up under mains power frequency. One of methods of setup uses periodic measurements of mains frequency. However the error of these measurements restricts reduction of spectrum leakage.

In the report results of a research of a voltage (current) spectrum measurement error because of a mains frequency measurement error are provided. Results are received by method of analytical and simulation modeling.

Keywords: power quality indicators, the discrete Fourier transform, spectrum leakage, mains frequency error.

1. Introduction

According to operating standards the measurement of many power quality indicators (PQI) are based on spectrum measurement, as a rule, by means of digital methods. Usually, a discrete Fourier transform (DFT) is used, since effective algorithms for its calculation have been developed, commonly referred to as a fast Fourier transform (FFT). Before the FFT, $N$ samples are accumulated in a measurement interval of $T_m$ duration, where $m$ is the number of periods of the main component ($m = 10$ for $50$ Hz power supply systems). The most efficient FFT algorithms require that $N$ be equal to the powers of two or four.

However, at a fixed sample rate $f_s$ and $N$, the deviation of the frequency of the mains $f_1$ from the nominal value causes spectrum distortions, often called spectrum leakage: the initial spectrum changes and additional components appear [1, 4]. To reduce the spectrum leakage, the value of $f_s$ should be adjusted to the frequency of the mains. To fulfill this requirement, hardware or software methods are used in practice.

The hardware methods are based on the use of a phase-locked loop oscillator, which performs the start of the ADC. In this case, the frequency $f_s$ tracks the frequency changes $f_1$. The main drawback of this approach is the presence of an additional error due to the dependence of the parameters of digital filters on the sampling frequency.

An alternative method involves collecting digital samples of signals with a constant sampling frequency $f_{s1}$, which is specified by a highly stable quartz oscillator. For this reason, the frequency characteristics of digital filters remain constant when the frequency $f_1$ is varied. To reduce the spectrum leakage, the filtered samples are preliminarily processed by a digital sampling rate converter (SRC), whose algorithm is implemented by software.

For SRC ratio control, periodic frequency measurements $f_1$ are used using an algorithm that is resistant to electrical network interference.

Based on the measured frequency value $f_1$, the sampling frequency $f_{s2}$ is calculated, ensuring the constancy of the number of samples $N$. Then, using the Lagrange interpolator included in the SRC, new values of the signal samples corresponding to $f_{s2}$ are calculated.

As a result, $N$ new samples are accumulated, which are used in the FFT in calculating the spectrum. The effectiveness of this method of tuning the sampling frequency is determined primarily by the error in measuring the frequency $f_1$.

The report considers the influence of the measurement error $f_1$ on the measurement error of the voltage (current) spectrum.

2. The error in measuring the spectrum of a sine-wave signal

Since the mathematical basis is the same for voltage and current, we will further use the generalized term signal.

The values of the discrete signal spectrum can be obtained by using a direct DFT [1]:

$$
\hat{X}[k] = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} kn},
$$

(1)
where \( \hat{x}[k] \) is the spectral component with the index \( k \); \( N \) is the number of samples; \( x[n] \) is the signal sample with the index \( n \).

Since the distortion of the spectral component will be characterized by the relative error or the harmonic coefficient, then in the analysis one can consider a sinusoidal signal with unit amplitude:

\[
x[n] = \sin(\omega n + \varphi),
\]

where \( \omega = 2\pi f / f_k \) is the angular frequency normalized to the sample rate \( f_s \); \( f \) is current frequency; \( \varphi \) is the initial phase.

Using the Euler identity, we represent \( x[n] \) in complex form. Then substituting \( x[n] \) into expression (1) and using the formula for the sum of the first \( N \) terms of the geometric progression, we get:

\[
\hat{x}[k] = \frac{1}{2j} \left[ e^{j\varphi} \left( 1 - e^{j\omega N} \right) - e^{-j\varphi} \left( 1 - e^{-j\omega N} \right) \right].
\]

Suppose that exactly \( N \) samples are placed on \( m \) signal periods with a frequency \( f_1 \). Then the following equalities hold:

\[
f_s = f_1N/m, \quad \omega = 2\pi f / f_s = 2\pi f m / f_1 N
\]

(4)

(5)

When equality (4) is absolutely satisfied and \( f = f_1 \), expression (3) gives the exact spectrum of the sinusoidal signal:

\[
\hat{x}[k] = \left\{\begin{array}{ll}
\frac{N}{2j} e^{j\varphi}, & \text{for } m = k; \\
0, & \text{for } m \neq k \text{ and } k \neq N - m.
\end{array}\right.
\]

(6)

In order to exclude the spectrum leakage with frequency deviation \( f_\delta \), equality (4) should be maintained by changing \( f_1 \) in proportion to \( f_\delta \). For this, it is required to periodically measure \( f_1 \). However, a finite measurement error \( f_\delta \) does not allow us to accurately perform equality (4) and, therefore, causes some distortion of the spectrum.

To analyze this distortion, we assume that

\[
\tilde{f}_1 = f_1(1 + \delta_f),
\]

(7)

where \( f_\delta \) is the result of the measurement \( f_1 \); \( \delta_f \) is relative measurement error \( f_1 \) (in relative units).

Then for \( f = f_1 \) and taking into account (7), expression (5) takes the form:

\[
\omega = \frac{2\pi m}{N(1 + \delta_f)} \approx \frac{2\pi m}{N} \left( 1 - \delta_f \right).
\]

(8)

Substituting expression (8) in place of \( \omega \) in (3), we obtain:

\[
\hat{x}[k] = -\frac{j}{2} e^{j\varphi} \times
\]

\[
\times \left[ \frac{1 - e^{-j2\pi \delta_f}}{1 - e^{j2\pi (m-k-m\delta_f)}} - \frac{e^{-j2\pi (1-e^{-j2\pi \delta_f})}}{1 - e^{j2\pi (m+k-m\delta_f)}} \right].
\]

(9)

We transform (9), bringing the numerators and denominators of fractions to a form that allows us to apply the Euler identity. As a result, we get:

\[
\hat{x}[k] = \frac{j}{2} e^{j\alpha} \times
\]

\[
\times \left[ \frac{\sin(\pi m \delta_f)}{\sin(\pi f (m-k-m\delta_f))} - \frac{e^{j\beta} \sin(\pi m \delta_f)}{\sin(\pi f (m+k-m\delta_f))} \right],
\]

(10)

where \( \alpha = \varphi - \pi (m \delta_f) - (m - k) / N \);

\[
\beta \approx 2\pi m \left( \frac{\delta_f}{N} + \frac{1}{N} \right) - 2\varphi.
\]

(11)

We define \( X[m] \) – the amplitude spectrum of the main component – as the modulus of expression (10) for \( k = m \). We first decompose the denominators of fractions into a Taylor series, restricting ourselves to one term:

\[
X[m] = \frac{N}{2} \left| \frac{\sin(\pi m \delta_f)}{-\pi m \delta_f} - \frac{e^{j\beta} \sin(\pi m \delta_f)}{2\pi m} \right|.
\]

(12)

Approximating the first term under the modulus sign by two terms, and the second by one term of the Taylor series, we get:

\[
X[m] = \frac{N}{2} \left| 1 + \frac{(\pi m \delta_f)^2}{6} - \frac{e^{j\beta} \delta_f}{2} \right|.
\]

(13)

Whence:

\[
X[m] = N/2 \times \sqrt{\left( \frac{(\pi m \delta_f)^2}{6} - \frac{\delta_f \cos \beta}{2} \right)^2 + \left( \frac{\delta_f \sin \beta}{2} \right)^2}.
\]

(12)

We simplify the last expression, performing its expansion in a Taylor series up to three terms:

\[
X[m] = N/2 \left( 1 - \frac{(\pi m \delta_f)^2}{6} + \frac{\delta_f \cos \beta}{2} \right).
\]

(12)

The characteristic error of the amplitude spectrum of the main component with frequency \( f_1 \) will be the relative error

\[
\delta_m = \frac{X[m] - N/2}{N/2}.
\]

(13)
We substitute in (13) for $X[m]$ its expression from (12). Then $\delta_m$ takes the form:

$$\delta_m = \frac{\delta_f \cos \beta}{2} - \frac{(\pi m \delta_f)^2}{6}.$$ \hfill (14)

At the zero initial phase, $\cos \beta \approx 1$. In this case, we obtain the approximation $\delta_m$ by a polynomial of the second degree:

$$\delta_m = \frac{\delta_f}{2} - \frac{\left(\pi m \delta_f\right)^2}{6}.$$ \hfill (15)

As follows from (14) and (15), $\delta_{m,\text{max}}$ — the maximum error in absolute value is equal to

$$\delta_{m,\text{max}} = -\left(\frac{\left(\pi m \delta_f\right)^2}{6} + \left|\frac{\delta_f}{2}\right|\right).$$ \hfill (16)

In figures 1 and 2 presents the results of simulation and analytical modeling using approximate formulas (14) – (16). Figure 1 shows the dependence of the error in measuring the amplitude spectrum of the sinusoidal signal $\delta_m$ from its initial phase $\varphi$ when measuring the frequency $f$ with an error of 0.03%.

Fig. 1. Dependence of error $\delta_m$ on the initial phase $\varphi$ at $\delta_f = 0.03\%$

The value of $\delta_{m,\text{max}}$ equal to 0.03 %, was chosen, proceeding from the requirements of the standard [2]. Other parameters of modeling: $N = 2048$; $f_1 = 52.5$ Hz; $m = 10$.

Figure 2 shows the dependence of the measurement error of the amplitude spectrum of the sinusoidal signal $\delta_m$ on the error in measuring the frequency $\delta_f$ at zero initial phase of the signal. The simulation parameters are similar to the parameters chosen in the construction of Fig. 1.

In figures 1 and 2 presents the results of simulation and analytical modeling using approximate formulas (14) – (16). Figure 1 shows the dependence of the error in measuring the amplitude spectrum of the sinusoidal signal $\delta_m$ from its initial phase $\varphi$ when measuring the frequency $f$ with an error of 0.03%.

As can be seen from Fig. 1 and Fig. 2, the results of analytical modeling and simulation are quite accurately matched. The simulations show that the relative deviation of the analytical dependences from the simulation results does not exceed approximately 0.028 %, while the error $\delta_f$ varies from -0.03 to 0.03 %.

Now we proceed to estimate the distortion of the spectrum: in the spectrum there appear components previously absent in the original signal. The distortion coefficients in this case will be the coefficients of the harmonics of the spectrum. To estimate these coefficients, it is necessary to find the modulus of expression (10) for an arbitrary $k$, but satisfying the condition: $\neq m$; $0 \leq k \leq N/2 - 1$, where $N$ is an even number.

Since the initial phase of the signal can take an arbitrary value, in the worst case, $e^{j\beta} = 1$. For small errors in measuring the signal frequency, expression (10) can be written in a simplified form:

$$X_k = \frac{1}{2} \left\{ \frac{\pi m \delta_f}{\sin(\pi (k - m)/N)} + \frac{\pi m \delta_f}{\sin(\pi (m + k)/N)} \right\},$$ \hfill (17)

where $X_k = |X[k]|$; $k$ is the number of the spectral component related to the frequency resolution (for $f_1 = 50$ Hz, the frequency resolution is 5 Hz); to determine the modulus of the harmonic with the number $h$, by formula (17), we must take $k = hm$. Then $K_h$ — harmonic coefficient with index $h$ — is defined by the formula:

$$K_h = \frac{X_k|k=hm}{N/2} 100\%.$$ \hfill (18)
Figure 3 shows the values of the harmonic coefficients that result from the spectrum leakage effect of the sinusoidal signal. The graph is constructed from the analytical dependences (17) - (18) for $\delta_f = 0.03\%$, $\varphi = 0$, $m = 10$.

Fig. 3. Harmonic coefficients due to the spectrum leakage of the sinusoidal signal

The graph in Fig. 3 practically coincides with the results of simulation at the same values of the parameters. It can be seen from Fig. 3 that the dependence makes it possible to determine the maximum value of the harmonic coefficients arising from spectrum leakage effect for an arbitrary initial phase of the signal. Regardless of the initial phase of the signal, the spectrum leakage effect leads to the greatest distortion of the second harmonic.

3. The measurement error of the polyharmonic signal spectrum

Let us consider the case of a polyharmonic signal consisting of the main component and $K$ harmonics (line spectrum). It is necessary to determine the distortion of the main component from the other components and the distortion of an arbitrary harmonic from the side of the main component and the remaining harmonics. A polyharmonic signal with a line spectrum in the time domain is described by the expression:

$$x[n] = \sum_{h=1}^{K} K_h \sin(\omega_h n + \varphi_h), \quad (19)$$

where $K_h$, $\omega_h$ and $\varphi_h$ are the coefficient (for the main component $K_1=1$), the angular normalized frequency and the initial phase of the harmonic with the index $h$.

The spectrum of a polyharmonic signal is represented as a sum of the spectra of individual components of the form (3). However, the resulting analytical expression of the spectrum, because of its cumbersomeness, is less convenient for investigating the distortion of the spectrum than simulation.

Figures 4 and 5 show the results of simulation. The simulation parameters are similar to the parameters chosen in the construction of Fig. 3. As the coefficients $K_h$ of the polyharmonic signal (19), the limiting values of the harmonic coefficients for the voltage from the standard are taken [3].

Figure 4 shows the contribution of each harmonic to the error in the amplitude spectrum of the main component. Figure 4 shows that the spreading effect of the spectrum of harmonics 2, 3, 5 most strongly affects the main component. When calculating the error of the amplitude spectrum of the voltage, it is necessary to take into account the influence of harmonics 2 through 7, 13 and 17. The influence of the other harmonics can be neglected. For the current, as analysis shows, the 3 and 5 harmonics are significant.

Figure 5 shows the errors in measuring the harmonic coefficients of a polyharmonic signal of the form (19), caused by the signal spectrum leakage effect. The modeling parameters are similar to the parameters chosen in the construction of Fig. 4.

Fig. 4. The effect of the harmonic spectrum leakage on the amplitude spectrum of the main component of the signal

The graph from above corresponds to the relative values of the error of the harmonics coefficients, whose coefficients are more than 1%. The graph below corresponds to the absolute values.
of the error of the harmonics coefficients, whose coefficients are less than 1%.

Fig. 5. Influence of a polyharmonic signal spectrum leakage on the amplitude spectrum of the harmonics of the signal

4. Conclusion

Based on the above material, we can draw the following conclusions:

1. To reduce signal spectrum leakage, it is necessary to adjust the sampling frequency to the mains frequency. One of the tuning methods uses periodic measurements of the mains frequency. However, the error of these measurements limits the reduction in the spectrum leakage.

2. The obtained analytical expressions (14) – (16) and (17), (18) allow us to estimate the maximum errors in measuring the amplitude spectrum of the sinusoidal signal by the DFT method, caused by an inaccurate measurement of the mains frequency.

3. Methods of analytical and simulation modeling found that to provide acceptable standards for the error in measuring the amplitude spectrum of the signal, it is sufficient to measure the frequency with an error of 0.03%.

5. References


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