DETERMINATION OF SAMPLE SIZE IN STATISTICAL PROCESS CONTROL WITH A SET RISK OF INCORRECT DECISION-MAKING

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Abstract: Control charts are a convenient tool for conducting SPC (Statistical Process Control), however, one should consider the risk of incorrect decision-making when using control charts to maintain the desired process level. The conditions associated with the process capability and sample size are considered for an efficient implementation of the SPC.

Key: Statistical methods, sampling control, quality management

1. Introduction

The main task of Statistical Process Control (SPC) is to ensure and maintain the process at an acceptable and steady level that will ensure the compliance of products with established requirements. Information obtained through SPC can also be useful in solving other tasks such as process comparisons, process development forecasts, adjustment and repair planning, etc. The main statistical tool used for Statistical Process Control is the control chart – a graphical tool for presenting and collating information obtained from sequential sample control and reflecting the current status of the process, with limits established on the basis of the inherent variation of the process (dispersion). The use of control charts and their careful analysis leads to the improvement of processes.

One of the most common types of control charts is the Shewhart chart. The primary purpose of this control chart is to assess whether the process is in a statistically steady state but the chart can also be used to maintain quality at a set level. The process is considered to be in a statistically steady state if the process is affected only by random (inherent to the process) causes. Under the influence of random causes only and normal distribution, the dispersion zone of the observed feature (with a probability of 99.73%) is $\pm 3\sigma$, and the Upper and Lower Control Limits (UCL and LCL) are set exactly at these levels. Exceeding these control limits is a signal for a disturbance (shift) in the process (statistically uncontrolled state), i.e. the process is not affected only by random causes but the shift is due to causes which need to be determined and the process corrected.

The control limits of the Shewhart chart are not related to Upper Specification Limit (USL) or Lower Specification Limit (LSL) but only to the desired (target) value X_0. This is an incentive for the manufacturer to maintain the process at the desired level.

2. Determining the Risk of Incorrect Decision Making

Given a desired (target) value X_0, known standard deviation of the process σ, and observed arithmetic mean of samples $\bar{X}$, the centerline of the chart is $CL = X_0$, and the control limits are $CL \pm 3\sigma_{\bar{x}}$ [4] [2].

Assuming a normal mean distribution, some 99.73% of the mean values will be within these limits, while some 0.27% will be outside these limits. Having any points outside the control limits due to purely random causes will be practically impossible. The risk of error of the first kind (mistakenly assuming a process shift) is:

α=2[1-NORMSDIST(3)]=0,0027 [2], which is practically negligible value. Such a low risk makes the control chart very resistant to any false signals of disturbances in the process.

Making an error of the second kind β (omitting an actual shift in the process), however, is a different thing. At an actual shift in the process $m\sigma$ with respect to X_0 to level X_1 (See Fig.1 below) the risk β will be:

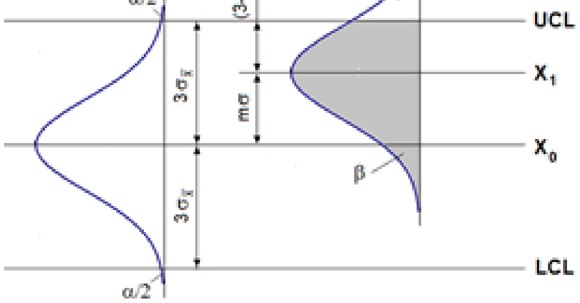

Fig.1

$$\beta = NORMSDIST(3 - m\sqrt{n}) \qquad (1)$$

where n is the sample size,
and the risk $\propto = 1 - \beta$.

For example, if a shift in the process away from the target value with one standard deviation (m=1) has occurred and the sample size is n=5, then β=0.777, i.e. the risk of this shift in the process not being reported is 77.7%. Such a low sensitivity to small shifts in the process away from the target value can also be viewed as an advantage of this type of control charts under certain conditions. One of these conditions is associated with the process capability defined by the index C_p=(USL-LSL)/6σ.

In order to make sure that the application of this type of control charts is efficient with respect to SPC, the following condition needs to be satisfied:

$$C_p \geq 1 + \frac{m}{3} \qquad (2)$$

Given a risk β and sample size n, we can determine m:

$$m = \frac{1}{\sqrt{n}}[3 - NORMSINV(\beta)]. \qquad (3)$$

For example when β =0.05 and n=5, m=2.07≅2, $C_p \geq 1 + \frac{m}{3} = 1.66$, i.e. if the process capability is C_p>1,66, then the risk β<5%.

For a process with good capability (large values of the index C_p) any small shifts of the process level relative to X_0=(USL-LSL)/2 will not result in non-compliant products.

3. Determining the sample size

Another factor on which the risk β will depend is the sample size n. This enables us to determine the sample size n, given a risk β and a known value of the index C_p, needed to ensure compliance with the requirements:

$$n \geq \left(\frac{3 - NORMSINV(\beta)}{3(C_p - 1)}\right)^2 \qquad (4)$$

Fig. 2 shows the relationship $n(C_p)$ at different risk β values.

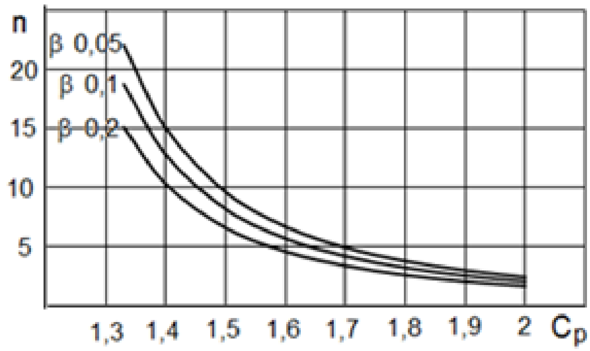

Fig.2

4. Conclusion

The Shewhart control charts are a convenient tool for conducting SPCs, but when used to maintain the desired process level, one should consider the risk of incorrect decision-making. The risk of not detecting an actual shift in process β may be determined using (1). If signaling with a low risk β is necessary for detecting a shift in the process level against the desired value which is less than mσ, then the process capability index C_p should satisfy the condition (2). At a given risk of omitting (not detecting) an actual shift in the process away from the desired level and a known value of the index C_p, the sample size should satisfy the condition (4). Satisfying these conditions will make the applicaion of SPC more efficient by using this type of control charts.

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5. References

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