COMPLIENCE PROBABILITY DETERMINATION ON BASIS OF THE MONTE CARLO METHOD

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Abstract: It is noted that, in contrast to the verification procedure used in the legal metrology, the verification procedure for calibrated measuring instruments has to take into account the uncertainty of measurements. The procedure of compliance probability determination on basis of Monte Carlo Method is considered. Example of calibration of vernier calliper is given.

Key-Words: probability of compliance, uncertainty in measurement, calibration, verification, uncertainty budget, maximal permissible error

1. Introduction

The item 5.6.1 of ISO/IEC 17025: 2005 [1] prescribes "All equipment used for tests and/or calibrations, including equipment for subsidiary measurements (e.g. for environmental conditions) having a significant effect on the accuracy or validity of the result of the test, calibration or sampling shall be calibrated before being put into service. The laboratory shall have an established programme and procedure for the calibration of its equipment."

The items 5.6.2.1.1 and 5.10.4.1.b of [1] recommend that calibration certificates for measuring instruments (MI) shall contain "the measurement results, including the measurement uncertainty and/or a statement of compliance with an identified metrological specification". From this requirement it follows that the presence of an indication of the conformity of the calibrated MI to the established metrological requirements or separate metrological characteristics is necessary in the calibration certificate.

The result of the measurement in the measuring instrument calibration is the systematic error of the indicating measuring instrument (IMI) or the assigned quantity value (indication) of the material measure (MM). When verification an IMI, the limit of the tolerance zone is usually its maximum permissible error (MPE). It should be noted that the main sources of uncertainty IMI calibration are: instrumental uncertainty of standard; its instability; changes in its operating conditions; mutual influence of the standard and IMI to be calibrated; the observed variation in the readings of the calibrated IMI; resolution of a displaying device IMI. With all uncertainty components taken into account, the extended measurement uncertainty during calibration may be greater than MPE. In this case its necessary evaluate the probability of compliance.

2. Probability of compliance

In documents [2,3] it is proposed to evaluate the probability of compliance of IMI by the formula:

$$p_c = \Phi_N\,[(MPE - |\widehat{\Delta}|)/u] = \Phi_N(z)\ , \qquad (1)$$

where $\Phi_N(z)$ – normal standard distribution function with variable *z*; $\widehat{\Delta}$, u – estimation of IMI's indication error and its standard uncertainty, respectively.

To find $\Phi_N(z)$, the [3] recommended to use the normalized normal distribution table. It's not only inconveniently, but nonapplicability in cases when the laws of distribution of calibration results are abnormally [4].

The formulas for normalized rectangular, triangular and trapezoidal distributions of variable are deduced in [6]. However, these models are only approximation of real law of distribution get in result of calibrations.

In those cases we recommend evaluate the probability of complains with help of Monte Carlo Method [6].

3. Monte Carlo Procedure

Monte Carlo procedure for construction of distribution function includes next operations, registered in uncertainty budget (table 1):

1. Recording the model equation:

$$\Delta = f(X_1, X_2, ..., X_N)\ , \qquad (2)$$

where $X_1, X_2, ..., X_N$ - input quantities (first column of the table 1).

2. Evaluation of the input quantities as $x_1, x_2, ..., x_N$ (second column of the table 1).

3. Evaluation of standard uncertainties of the input quantities as $u(x_1), u(x_2), ..., u(x_N)$ (third column of the table 1).

4. Assigning the probability density functions (PDF's) for input quantities (fourth column of the table 1).

5. Selecting the number M of Monte Carlo trials to be made ($M \geq 10^4$).

6. Generating *M* trials of measurand for vectors, by sampling from the assigned PDFs, as realizations of the (set of *N*) input quantities X_i.

7. For each such vector, forming the corresponding model value of Δ, yielding *M* model values Δ_i.

8. Calculate an estimate $\overline{\Delta}$ of Δ by the formula:

$$\overline{\Delta} = \frac{1}{M} \sum_{i=1}^{M} \Delta_i \quad . \qquad (3)$$

9. Calculate unbiased estimate Δ_i^* using the formula:

$$\Delta_i^* = \Delta_i - \overline{\Delta} \quad . \qquad (4)$$

10. Sort these *M* model values Δ_i^* into strictly increasing order, using the sorted model values to provide an implementation of the propagation of distributions *G*.

11. Construction the dependence

$$G = \varphi(100 \cdot i/M) \ ,$$

which correspond to dependence:

$$p_c = \varphi(\text{MPE} - |\overline{\Delta}|) \quad . \qquad (5)$$

12. For specified MPE and $\overline{\Delta}$ finding probability of compliance p_c as the value of column $100 \cdot i/M$ corresponding the value of column MPE-|Δi|.

4. Example

Calibration of a vernier calliper with a resolution of 0,05 mm [4].

The model equation for this example is:

$$\Delta = l_{iX} - l_S + L_S \cdot \overline{\alpha} \cdot \Delta t + \delta l_{iX} + \delta l_M \ , \qquad (6)$$

where: Δ – error of indication of the calliper; l_{iX} – indication of the calliper; l_S – length of the actual gauge block,

$$\hat{l}_S = L_S \pm 0,8\ \mu m = 150\, mm \pm 0,8\ \mu m \ ;$$

L_S – nominal length of the actual gauge block;

$\overline{\alpha}$ =11,5·10^{-6}°C^{-1} - average thermal expansion coefficient of the calliper and the gauge block;

Δt – difference in temperature between the calliper and the gauge block, with limits ±2 °C;

δl_{iX} – correction due to the finite resolution of the calliper, with limits ±25 μm;

δl_M – correction due to mechanical effects, such as applied measurement force, Abbe errors, flatness and parallelism errors of the measurement faces, with total limits of error ± 50 μm.

All input quantities have rectangular distributions.

Realization of the points 6-8 of described above Monte Carlo procedure, give the dependence $p_c(\text{MPE} - |\Delta|)$ represented on the fig.1.

Table 1. Uncertainty budget

Input quantity	Estimate	Standard uncertainty	Probability distribution	Sensitivity coefficient	Uncertainty contribution
X_1	x_1	$u(x_1)$	PDF 1	c_1	$u_1(y)$
X_2	x_2	$u(x_2)$	PDF 2	c_2	$u_2(y)$
:	:	:	:	:	:
X_N	x_N	$u(x_N)$	PDF *N*	c_N	$u_N(y)$
Measurand	Estimate	Combined standard uncertainty	Coverage probability	Coverage factor	Expanded uncertainty
Δ	$\hat{\Delta}$	$u_c(y)$	0,95	k	U

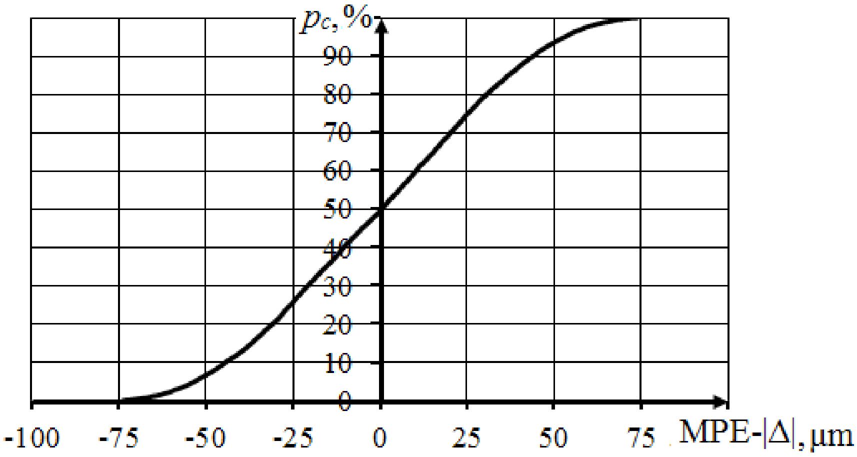

Fig. 1. Dependence of

For described above data's the error of indication of the caliper is 100 µm; combined standard uncertainty is 33 µm and expanded uncertainty U is 59 µm.

From fig.1. follow:

- In example [4] E_X =100 µm. For this error $\text{MPE}-|\Delta|$ =- 50 µm, p_c =7 % and vernier calliper is unfit for perform measurements.
- When Δ =50 µm (by results of verification vernier calliper is valid), $\text{MPE}-|\Delta|$ =0 µm and p_c =50 %.
- When Δ =25 µm, $\text{MPE}-|\Delta|$ =25 µm and p_c =75 %.
- When Δ =0 µm, $\text{MPE}-|\Delta|$ =50 µm and p_c =93 %.

5. Conclusions

All probabilities of compliance less than 95%. It should be noted that in all cases the condition *U <MPE/3*, given in [3], is not observed. Practice shows that the number of callipers with =50 µm is about 40% of those arrived at the test.

6. References

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