

## ESTIMATION OF UNCERTAINTY IN CALIBRATION OF INDUCTANCE MEASURES BY USING THE STATE PRIMARY STANDARDS OF THE UNITS OF ELECTRICAL CAPACITANCE, INDUCTANCE AND DISSIPATION FACTOR

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*Abstract:* The main points of estimation of uncertainty in calibration of inductance measures by using the State primary standards of the units of electrical capacitance, inductance and dissipation factor are considered in the article.

*Keywords:* standard, the inductance measure, total standard uncertainty, expanded uncertainty.

### Introduction

In 2001-2009, the State primary standards of electrical capacitance units, inductance units and dissipation factor were created and successfully are used in State Enterprise "All-Ukrainian State Scientific and Production Centre for Standardization, Metrology, Certification and Protection of Consumer" (SE "Ukrmetrteststandard").

The standards are based on the universal automated precision comparator [1], which allows to realize the calibration in the total range of values towards both: high impedance with the basis on the low impedance, capacitance with the basis on the resistance, inductance with the basis on the capacitance.

Similar standards were created in SE "Ukrmetrteststandard" for such national metrology institutes as BelGIM (Belarus), GUM (Poland) and NIST (USA).

The aim of the article – the main points of estimation of uncertainty in calibration of inductance measures based on a modern approach to the processing of measurement results by using the State primary standards of the units of electrical capacitance, inductance and dissipation factor must be provided.

### 1. Basic Points

To calibrate high-precision inductance measures, traceability from a calibrated high-precision measure of capacity of 100 pF or 10 pF with the appropriate uncertainty should be ensured.

The State primary standard of the units of electrical capacitance and dissipation factor DETU 08-06-01 includes the group of four precision capacitance measures Andeen-Hagerling (USA)

AH11A ( $4 \times 100$  pF and  $4 \times 10$  pF). Due to the calibration of the specified measures conducted at NIST (USA), PTB (Germany) and NPL (Great Britain), as well as own constant research, the value of the electrical capacitance of all capacitance measures AH11A is known with the expanded uncertainty  $U(C_{AH})=7.4 \cdot 10^{-6}$  pF with the probability at the coverage factor .

The working frequencies of the standard are 1 kHz and 1.592 kHz. The calibration of capacitance measure AH11A on the frequencies referred above was conducted on national standards of other countries, which are based on the use of fundamental physical constants.

The transmission of the unit size of inductance over the value range is carried out with the use of the universal automated precision comparators included in the standards [2].

The comparators has the following transmission ratio values: 1:1 or 1:10. Using these transmission (comparison) ratio values only, it is possible to realize the transmission of the unit size of inductance from capacitance measures by consecutive calibrations in the total range of values toward both high and low impedance.

An example of transmission of the unit size of inductance over the value range from capacitance measures in the calibration of inductance measure with the nominal value of 1  $\mu$ H based on the standard capacitance measure of 100 pF and with the use of a series intermediate standard thermostated capacitance and inductance measures is represented in Fig. 1.

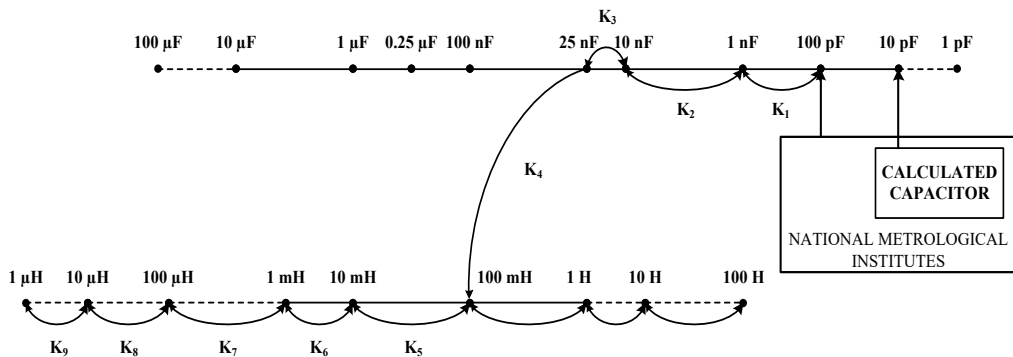


Fig. 1. The transmission of the unit size of inductance over the value range with the basis on the calculable capacitor

## 2. The estimation of uncertainty in the calibration of inductance measures

The model (equation) of measurements follows from the measurement scheme:

$$L_X = \frac{K_5 K_6 K_7 K_8 K_9}{\omega^2 K_1 K_2 K_3 K_4 C_{100\text{pF}}}, \quad (1)$$

where:

$$C_{100\text{pF}} = C_S + \Delta C_{\text{TS}} + \Delta C_{\text{FS}} + \Delta C_{\gamma S}, \quad (2)$$

$K_1$  is the transmission coefficient of the comparator in the calibration of the capacitance measure 1 nF with the basis on the capacitance measure AH11A with the nominal value of 100 pF:

$$K_1 = \frac{C_{1\text{nF}}}{C_{100\text{pF}}}; \quad (3)$$

$K_2$  is the transmission coefficient of the comparator in the calibration of the capacitance measure 10 nF with the basis on the intermediate capacitance measure with the nominal value of 1 nF, for the box of temperature-stabilized capacitance measures CA 5200RC:

$$K_2 = \frac{C_{10\text{nF}}}{C_{1\text{nF}}}; \quad (4)$$

$K_3$  is the transmission coefficient of the comparator in the calibration of the capacitance measure 25 nF with the basis on the intermediate

capacitance measure with the nominal value of 10 nF, for the box of temperature-stabilized capacitance measures CA 5200RC:

$$K_3 = \frac{C_{25.33\text{nF}}}{C_{10\text{nF}}}; \quad (5)$$

$K_4$  is the transmission coefficient of the comparator in the calibration of the inductance measure 100 mH with the basis on the intermediate capacitance measure 25.33 pF on the frequency  $\omega$ ;

$\omega$  is the operating frequency of the test signal 6279.897 rad/s (1 kHz) on which measurements are made;

$K_5$  is the transmission coefficient of the comparator in the calibration of the inductance measure 10 mH with the basis on the intermediate inductance measure 100 mH:

$$K_5 = \frac{L_{10\text{mH}}}{L_{100\text{mH}}}; \quad (6)$$

$K_6$  is the transmission coefficient of the comparator in the calibration of the inductance measure 1 mH with the basis on the intermediate inductance measure 10 mH:

$$K_6 = \frac{L_{1\text{mH}}}{L_{10\text{mH}}}; \quad (7)$$

$K_7$  is the transmission coefficient of the comparator in the calibration of the inductance measure 100  $\mu\text{H}$  with the basis on the intermediate

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inductance measure 1 mH:

$$K_7 = \frac{L_{100\mu\text{H}}}{L_{1\text{mH}}}, \quad (8)$$

$K_8$  is the transmission coefficient of the comparator in the calibration of the inductance measure 10  $\mu\text{H}$  with the basis on the intermediate inductance measure 100  $\mu\text{H}$ :

$$K_8 = \frac{L_{10\mu\text{H}}}{L_{100\mu\text{H}}}; \quad (9)$$

$K_9$  is the transmission coefficient of the comparator in the calibration of the inductance measure 1  $\mu\text{H}$  with the basis on the intermediate inductance measure 10  $\mu\text{H}$ :

$$K_9 = \frac{L_{1\mu\text{H}}}{L_{10\mu\text{H}}}; \quad (10)$$

$C_{100\text{pF}}$  is the actual value of the Andeen-Hagerling AN11A capacity measure with the nominal value of 100 pF, taking into account the drift of the main parameter, as well as other influential factors;

$C_S$  is the value of the Andeen-Hagerling AN11A capacity indicated in the calibration certificate;

$\Delta C_{TS}$  is correction for the temperature dependence of the capacitance measure AH11A;

$\Delta C_{fS}$  is correction for the frequency dependence of the capacitance measure AH11A;

$\Delta C_{\gamma S}$  is correction for the drift of the measure AH11A since the last calibration.

It should be noted that, the calibration of inductance measures must be carried out either for 2- and 3-terminals, in accordance with the requirements of the customer.

The example of the uncertainty budget of measurements of the capacitance value in the calibration of the capacitance measure AH11A  $C_X$  is presented in Table 1 [2].

The example of the uncertainty budget of measurements of the inductance value in the calibration of the inductance measure  $L_X$  with the basis on the capacitance measure AH11A  $C_{100\text{pF}}$  is presented in Table 2.

Calculation of the relative total standard uncertainty  $\omega(L_X)$  and relative expanded uncertainty  $W(L_X)$  in the transmission of the unit size of the physical quantity from the capacitance measure AH11A with the nominal value of 100 pF to the calibrated inductance measure  $L_X$  with the nominal value of 1  $\mu\text{H}$  is carried out in a relative form by the formulas (11, 12):

$$\omega(L_X) = \sqrt{\omega^2(C_{100\text{pF}}) + \sum_{i=1}^N p_i^2 \omega_i^2(x_i)} \quad (11)$$

$$W(L_X) = k\omega(L_X) \quad (12)$$

*Table 1. The measurement uncertainty budget in the calibration of the capacitance measure AH11A*

Quantity $X_i$	Estimate $x_i$	Standard uncertainty $u(x_i)$ , pF	Probability distribution	Method of evaluation (A, B)	Sensitivity coefficient $c_i$	Uncertainty contribution $c_i u(x_i)$ , pF
$C_S$	100,00002	3,7E-05	normal	A	1	3,7E-05
$\Delta C_{TS}$	0	7,1E-07	normal	B	1	7,1E-07
$\Delta C_{fS}$	0	4,1E-07	normal	B	1	4,1E-07
$\Delta C_{\gamma S}$	-1,10E-06	5,8E-07	normal	B	1	5,8E-07
$C_{100\text{pF}}$	100,0000189					
Combined standard uncertainty, pF					$u(C_X)$	3,70E-05
Effective degrees of freedom					$\nu_{\text{eff}}$	>200, $k = 2$
Expanded uncertainty ( $p \approx 95\%$ )					$U(C_X)$	<b>7,403E-05</b>

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*Table 2. The measurement uncertainty budget in the calibration  
of the inductance measure  $L_x$  with the basis on the capacitance measure AH11A  $C_{100pF}$*

Quantity $X_i$	Estimate $x_i$	Relative standard uncertainty $\omega(x_i) = u(x_i)/ x_i $	Probability distribution	Method of evaluation (A, B)	Sensitivity coefficient $p_i$	Relative uncertainty contribution $p_i\omega(x_i)$
$C_{100pF}$	100,000019 pF	3,83E-07	normal	B	-1	-3,83E-07
$\omega$	6283,185370 rad/c	1,00E-07	normal	B	-2	-2,00E-07
$K_1$	10,000245	1,20E-06	normal	A	-1	-1,20E-06
$K_2$	10,000239	1,20E-06	normal	A	-1	-1,20E-06
$K_3$	2,532112	1,50E-06	normal	A	-1	-1,50E-06
$K_4$	10,100035	7,00E-06	normal	A	-1	-7,00E-06
$K_5$	0,099970	1,50E-06	normal	A	1	1,50E-06
$K_6$	0,100041	1,50E-06	normal	A	1	1,50E-06
$K_7$	0,099982	2,00E-06	normal	A	1	2,00E-06
$K_8$	0,099999	3,00E-06	normal	A	1	3,00E-06
$K_9$	0,100001	5,00E-06	normal	A	1	5,00E-06
$L_x$	0,990337 $\mu$ H					
Relative combined standard uncertainty					$\omega(L_x)$	9,84E-06
Effective degrees of freedom					$n_{eff}$	>200, $k = 2$
<b>Relative expanded uncertainty (<math>p \approx 95\%</math>)</b>					<b><math>W(L_x)</math></b>	<b>3,46E-05</b>

In the sense of the standard uncertainty of the coefficients  $K_1, K_2, \dots, K_9$ , the following is taken account: the deviation due to the error of the quantization of the comparator; correction for the sensitivity of the comparator and the error of comparison. The values of the coefficients  $K_1, K_2, \dots, K_9$  are given in the passport of the comparator, but can be specified for each point of the measurement range by comparing pre-calibrated high-precision measures. It should be noted that in the relative total standard measurement uncertainty of the calibration result, it is also necessary to take into account the frequency dependence of the transmission factor of the comparator. Frequency instability has a significant effect on the result of measurements when calibrating both the capacity measurements and the measures of inductance. However, during the measurements, the frequency drift is negligible and the measurement uncertainty

is about the value of  $1 \cdot 10^{-10}$ . Thus, the components of uncertainty introduced by the frequency dependence can be neglected.

The measured value of inductance of the measure with the nominal value of  $1 \mu\text{H}$  at the measurement temperature of  $(22 \div 24)^\circ\text{C}$  and relative humidity of  $(30 \div 45)\%$  at the frequency of the examined signal of  $1 \text{ kHz}$  made up  $0.990337 \mu\text{H} \pm 3.46 \cdot 10^{-5}$ .

This measurement result corresponds to the published CMC-lines in the table of calibration and measurement capabilities of State Enterprise "Ukrmetrteststandard" on the website of KCDB BIPM.

### Conclusions

The developed calibration procedure allows to estimate the relative total standard and relative extended measurement uncertainty when calibrating the reference inductance measures with using the State primary standards of the units of electrical

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capacitance, inductance and dissipation factor. The use of this calibration procedure allows to estimate the measurement uncertainty of the inductance value of standard measures within the framework of international comparisons.

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