SOME FEATURES AND OPPORTUNITIES FOR CALIBRATION OF ANALYZERS OF ELECTRIC POWER BY HARMONICS

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Abstract: The possibility for evaluation of the quality of electrical energy for the purpose of calibration of analyzers is considered by using a periodic square-wave pulse signal instead of the traditionally used harmonic signals. The square-wave pulse signal is represented in Fourier series, as each harmonic component is a function of the amplitude and the impulse signal duty ratio. Thus, its harmonic component is defined by strictly determined parameters much better than the parameters of a generated harmonic signal. A metrological analysis of the correction of the generated by the standard calibrator, values of harmonic voltages and currents has been performed.

Key-Words: analyzer of electric power, calibration, harmonic.

1. Introduction

The quality of the electrical energy is a matter of essential importance to the economy, which engages daily the attention of professionals in the areas of production, transmission, distribution and final consumption. To a considerable extent, the quality of the electrical energy depends on the characteristics of the electrical parameters: frequency, amplitude and shape of the voltage and symmetry of the three-phase voltage. Electricity analyzers are the measurement devices that provide an adequate assessment of the quality of the electrical energy. These measuring instruments measure the values of more than ten electrical quantities and parameters of the electrical energy, process the collected measurement information and provide a quality estimate according to the established normative basis.

The development of the technique and technology in economies, related to the creation and use of devices and facilities, sensitive to the described disturbances of the electricity network, requires adequate maintenance and traceability of the metrological characteristics of the power analyzers. This is achieved by calibrating of these devices with respect to the relevant quantities and parameters.

2. Calibration by harmonics and mathematical model

In calibrating of the electrical energy analyzer, referred further as the analyzer, the method of comparison of the measuring instrument with the traceable standard (reference calibrator, shortly - a calibrator), corresponding to the requirements of the traceability chain, according to the block diagram of fig.1 is applied.

The calibration process determines the actual effective values of the corresponding harmonics of voltage or current, in conformation with the valid standards and norms or according to values indicated by the corresponding laboratory. In the calibration the relationship between the reading of analyzer and the realized by calibrator harmonic voltage or current is established.

The mathematical model for the estimate of the actual effective value of the voltage *n*-th harmonic, according to [1] is the following:

$$U_{n,act} = U_{n,cal} - \delta U_{n,et} + \delta U_{n,res.cal}, \qquad (1)$$

where:

- $U_{n,cal}$ is the measured effective value of the harmonic voltage (voltage of *n*-th harmonic) obtained by the calibrated analyzer, using in repeated measurements the estimate $U_{n,cal} = \frac{1}{k}\sum_{i=1}^{k} U_{n,cal,i}$, determined as an average value of the individual observations (measurements) $U_{n,cal,i}$;
- k - a number of measurements usually selected $k \geq 10$;
- i - a measurement index;
- $\delta U_{n,res.cal}$ - a correction of the measured value of the harmonic voltage, due to the resolution of the calibrated analyzer;
- $\delta U_{n,et}$ - a correction of the set by the calibrator harmonic voltage, which is generally due to different reasons such as:

- $\delta U_{n,s.et}$ - a deviation of the set (generated)

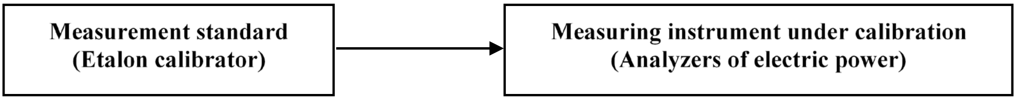

Fig.1. Scheme of calibration

value of the calibrator due to combined effects by offsets, non-linearity, and other instrumental and methodic imperfections of the calibrator. This deviation can be determined from the calibrator technical documentation (while the calibrator is not calibrated) or from its calibration certificate as a correction of the calibration point ;

- $\delta U_{n,dr.et}$ - a drift of the generated by the calibrator value, compared to the calibrator's last calibration (a drift since its last calibration);

- $\delta U_{n,t.et}$ - a deviation of the calibrator value as a result from a change in the environment temperature;

- $\delta U_{n,l.et}$ - a deviation of the calibrator value due to changes in the supply voltage;

- $\delta U_{n,z.et}$ - a deviation of the calibrator as a result from the energy exchange of the calibrator due to the input impedance of the calibrated analyzer.

In calibration of a particular analyzer only these components should be considered which have the most significant contribution to the correction $\delta U_{n,et}$.

The deviation of the calibrator set point value of the harmonic voltage of the calibrator $\delta U_{n,s.et}$ is determined by the relationship

$$\delta U_{n,s.et} = \delta_{Un,et} U_n , \qquad (2)$$

where

- U_n is the effective value of the harmonic voltage set by the calibrator;

- $\delta_{Un,et}$ is the relative error of the calibrator.

Similarly, the mathematical model for estimating of the actual effective value I_n of the *n*-th harmonic current according to [1] has the form:

$$I_{n,act} = I_{n,cal} - \delta I_{n,et} + \delta I_{n,res.cal} , \qquad (3)$$

where the individual components for the harmonic currents in the expression are analogous to the components of harmonic voltages in (1).

In calibration of the analyzer using a voltage harmonic by the help of the calibrator, the nominal effective voltage value of the main (first) harmonic $U_{1nom} = 230V$ and an effective value for the corresponding (relevant) harmonics U_n, for which the analyzer is calibrated, are set. This approach can be applied only if the calibrator has the ability to generate a periodic signal, which is the sum of two or more harmonic signals, one of which is the voltage of the main harmonic with a frequency $f = 50Hz$. If the calibrator does not have such a possibility, in the calibration of the analyzer it is suggested to use a periodic squarewave bipolar signal.

The calibration of the analyzer of the electric power based on a current harmonic is analogous to the calibration of voltage harmonics. The only difference is the use of relationship (3) instead of (1), therefore, in the present paper considers only the process, the features and the possibilities for calibration of voltage harmonics.

3. Calibration of harmonics using a periodic squarewave bipolar signal

In this case, the calibrator sets the effective value $U_{n,et}$ of each harmonic by voltage through a squarewave bipolar signal with a basic frequency $f = 50Hz$, which does not include a DC component. Two parameters are set: amplitude value and a duty cycle μ of the squarewave pulse signal in order to obtain the desired effective value $U_n = \frac{U_{nm}}{\sqrt{2}}$ of the respective harmonic according to the norms specified in standards EN50160 or EN61000-4-7, or the value, set by the laboratory. For the purpose of obtaining a specific effective value for the particular harmonic the squarewave pulse signal is decomposed in Fourier series.

3.1. Presentation of a periodic complex signal through a trigonometric Fourier series

One of the approaches to analyzing real periodic complex (nonharmonic) signals is their decomposition in Fourier series [2,3]. In this way every complex periodic signal $x(t)$ is represented as a sum of elementary harmonic signals, which are called harmonic components, or harmonics or harmonic components of the complex periodic signal. The complete trigonometric form of the Fourier series of the complex periodic signal has the form:

$$x(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left[A_n \cos\left(\frac{2n\pi t}{T}\right) + B_n \sin\left(\frac{2n\pi t}{T}\right)\right] \quad (4)$$

where

$$A_0 = \frac{2}{T}\int_0^T x(t)dt$$

$$A_n = \frac{2}{T}\int_0^T x(t)\cos\left(\frac{2n\pi t}{T}\right)dt = \frac{2}{T}\int_0^T x(t)\cos n\omega_1 t dt$$

$$B_n = \frac{2}{T}\int_0^T x(t)\sin\left(\frac{2n\pi t}{T}\right)dt = \frac{2}{T}\int_0^T x(t)\sin n\omega_1 t dt$$

Here ω_1 is the angular frequency of the main harmonic, with *n* is denoted the *n*-th harmonic number, A_n and B_n are the amplitudes of the two harmonic constituents with multiple frequencies, and $\frac{A_0}{2}$ is the constant component of the signal $x(t)$. The relationship between the angular frequency ω_1, the main frequency *f* and the period *T* of the signal $x(t)$ is $\omega_1 = 2\pi f = \frac{2\pi}{T}$ and $f = \frac{1}{T}$.

3.2. Presentation of a periodic squarewave bipolar signal through a trigonometric Fourier series

The periodic squarewave pulse signal is one of the most common reference signals. In Figure 2 the shape of a periodic squarewave bipolar pulse signal (voltage) $u(t)$ is shown. It is described by the expression:

$$u(t) = \begin{cases} +U_m & t \in 0,\tau \\ -U_m & t \in \tau, T \end{cases} \quad (5)$$

where

- U_m - is the amplitude of the impulse,
- τ - is the duration of impulse,
- T - is the period of the pulse signal.

An important parameter of the periodic pulse signal is its duty cycle μ, which depends on the duration of the impulse and the period of the pulse signal and is determined by the expression $\mu = \frac{\tau}{T}$.

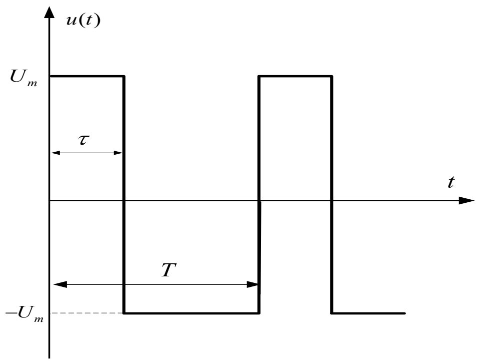

Fig.2. Timing diagram of periodic bipolar squarewave signal

By applying the decomposition of the periodic bipolar squarewave signal in Fourier series according to (4), the following expression is obtained:

$$x(t) = 2U_m(\mu - 0,5) + \frac{4U_m}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi\mu)}{n}\cos\left(\frac{2n\pi t}{T}\right) \quad (6)$$

3.3. Setting an amplitude value of the harmonic for which the analyzer is calibrated

If in calibration of the analyzer an effective value U_n for its *n*-th harmonic has to be set, its amplitude (maximum) value is $U_{nm} = \sqrt{2}U_n$. Let's assume that an analyzer is calibrated on one single harmonic, for example the *n*-th harmonic. Then the signal, which is generated by the calibrator and which is submitted to the input of the calibrated analyzer contains the sum of the basic and the *n*-th harmonic, as their amplitudes are the following:

- the amplitude of the basic harmonic

$$U_{1m} = \frac{4U_m}{\pi}\sin(\pi\mu) = \sqrt{2}U_1 = 325,269V \quad (7)$$

as $U_1 = U_{1nom} = 230V$ is the effective value of the basic harmonic, which usually is nominal phase voltage and

- the amplitude of the *n*-th harmonic

$$U_{nm} = \frac{4U_m}{n\pi}\sin(n\pi\mu) \quad (8)$$

The effective value of the *n*-th harmonic is

$$U_n = \frac{U_{nm}}{\sqrt{2}} = \frac{2\sqrt{2}U_m}{n\pi}\sin(n\pi\mu) \quad (9)$$

Equations (7) and (8) form a system from which the two variables U_m and µ can be determined, in case values for the harmonic number *n*, the amplitude of the harmonic U_{nm} and the amplitude of the main harmonic U_{1m} are previously assigned.

An analytical solution of the system as a mathematical formula has not been found yet. In this case, it is proposed to use numerical determination of the values for both variables U_m and µ, such that desired effective values of the main harmonics U_1 and the corresponding harmonic U_n, e.g. Standard EN50160, are realized. For this purpose, it is appropriate to use the program product Wolfram Mathematica [4] or other similar.

3.4. Analysis of error in calibration using harmonics by periodic squarewave pulse signal

Each harmonic has an effective value represented by (9), which is a function of the magnitude U_m and µ. These two magnitudes are set by the calibrator with relevant errors, which results in an error of the set effective value of the corresponding harmonics. For the analysis of this error it is necessary to find the full differential of function (9), which has the form

$$dU_n = \frac{\partial U_{nm}}{\partial U} dU_m + \frac{\partial U_{nm}}{\partial \mu} d\mu$$

$$\Delta U_n = \frac{2\sqrt{2}}{n\pi}\sin(n\pi\mu)\Delta U_m + 2\sqrt{2}U_m\cos(n\pi\mu)\Delta\mu \qquad (10)$$

Thus, for the relative error of the set effective value of the *n*-th harmonic it is obtained

$$\delta_{Un,et} = \frac{\Delta U_n}{U_n} = \frac{\Delta U_m}{U_m} + n\pi\mu.\mathrm{ctg}(n\pi\mu)\frac{\Delta\mu}{\mu} = \\ = \delta_{Um} + n\pi\mu.\mathrm{ctg}(n\pi\mu)\delta_\mu = \delta_{Um} + a_\mu\delta_\mu \qquad (11)$$

where δ_{Um} is the relative error of the amplitude and δ_μ is the relative error of the duty cycle of the squarewave pulse signal.

In this case, interesting is the study of the influence factor function $a_\mu = a_\mu(\mu) = n\pi\mu.\mathrm{ctg}(n\pi\mu)$ for $0 < \mu < 1$, which is graphically represented in Figure 3 for the first and third and in Figure 4 for the second and fourth harmonics. It is seen that for each particular harmonic, the quantity a_μ is reset at certain values of µ, as for the first harmonic there is

$$\delta U_{n,s.et} = \delta_{Un,et}U_n = \left\{\delta_{Um} + \left[n\pi\mu.\mathrm{ctg}(n\pi\mu)\right]\delta_\mu\right\}\left[\frac{2\sqrt{2}U_m}{n\pi}\sin(n\pi\mu)\right] \qquad (12)$$

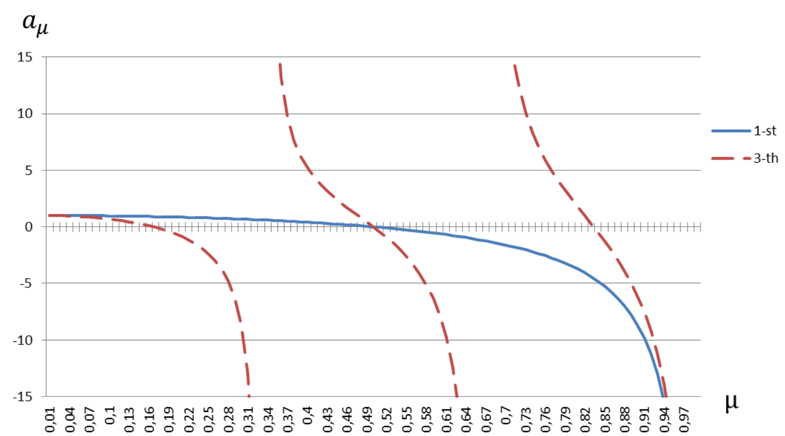

Fig.3: Relationship $a_\mu(\mu)$ for 1-st and 3-th harmonic

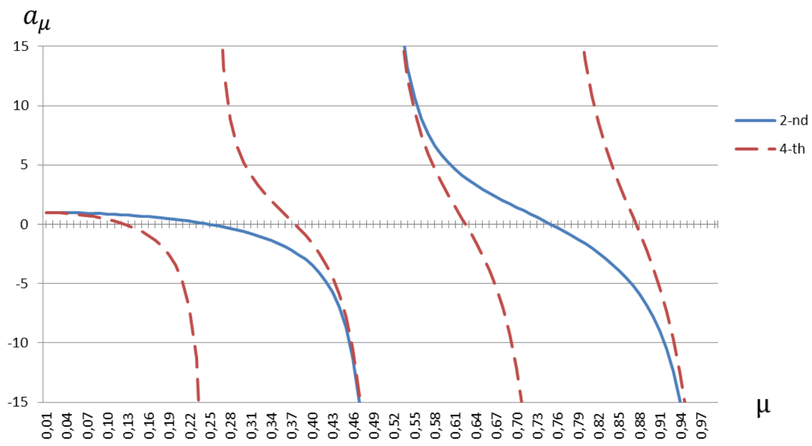

Fig.4: Relationship $a_\mu(\mu)$ for 2-nd and 4-th harmonic

one such value, for the second - these values are two, for the third - three, etc. At these points the relative error of the set effective value of n-th the harmonics do not depend on the value of the relative error of the duty cycle of the squarewave impulse signal, which should be taken into account when realizing the value of a particular harmonic.

Using the expression (11) and the relationship (2) for the deviation of the reference calibrator effective value of the n-th harmonic it is obtained

4. Conclusions:

1. The method for calibration by harmonics using a periodic bipolar squarewave signal is appropriate when the reference calibrator can't generate two or more harmonic signals.

2. The error with which the amplitude U_m of the periodic squarewave bipolar signal is set directly participates in the total error of setting the effective value of harmonic voltages.

3. The values of the duty cycle μ should be should choose such, where the influence factor a_μ to reset, i.e. to exclude the influence of a relative error of the duty cycle.

4. The values of the amplitude of the squarewave impulse U_m and duty cycle μ of the periodic bipolar squarewave signal should be chosen so as to ensure both: nominal value of the main harmonic and wanted nominal harmonic value for which the analyzer is calibrated. This is not always possible because the amplitude of the set periodic squarewave pulse signal is limited. Then these two parameters are chosen so as to realize the desired effective value of the corresponding harmonics at the greatest possible value of the main harmonic.

5. References

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