FORMATION OF THE RULE DECISION-MAKING ABOUT SUITABILITY PRODUCTS ON THE BASIS OF THE ADAPTIVE ALGORITHM

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Abstract: It is shown that the results of the evaluation of the object of inspection parameters are inevitably associated with uncertainty due to imperfections in this procedure, equipment and the influence of external random factors. Reduction of this influence is especially important within certain limits around the limits of the tolerance interval while reducing the number of additional measurements. Therefore, the decisive rule on the suitability or unsuitability of the inspection object should take into account both the possible uncertainty of the result and the requirements for the cost and complexity of the verification procedure, as well as the specified reliability. It is shown that the application of the algorithm makes it possible to increase the probability of making the correct decision about compliance for each object under test and, in comparison with known methods, does not significantly affect the decrease in productivity and increase the cost price of the product. The practical use of the algorithm showed that 5 additional stages of the study make it possible to reduce the probability of false decisions by at least 3 times, while the increase in control operations does not exceed 60%.

Keywords: conformity assessment, an uncertainty of measurement, adaptive successive algorithm, probability, decision making.

1. Introduction

In many cases, conformity assessment means inspection as a procedure to establish that the product meets the specified requirements, based on the measurement results. For the measured value, the requirements are set limits of the tolerance interval, which separate the intervals of admissible values of the measured value from the critical (inadmissible) [1]. An object is responsible for the given requirements when the value of its property (parameter) is located within the tolerance interval. Indications measuring system [2] reflect information on the value of the tested quantity by means of a model of measurement, which includes effects of both systematic and random variables (or their totality) that cause the uncertainty of the result. Due to the uncertainty of measurement, there is always a risk (probability) of making an erroneous decision about the conformity or non-conformity of an object (its parameter) installed requirements on the basis of the measured value of the property of the object (its parameter). Thus, comparing the coverage interval with tolerance interval is the basis for making a decision on the correspondence of the object, while making decisions with the highest probability. The need to increase the likelihood of a decision also emphasizes the presence of a number of normative documents that establish methods for increasing the probability of making a decision on the results of measuring the parameter of the monitoring quantity, taking into account the uncertainty associated with it [3-6].

2. Formulation of the problem

To enhance the probability of result during measurement at inspection is traditionally used by more precise measuring systems; carry out multiple observations monitoring quantity, taking the average value for the result of the measurement; set tolerance intervals (guard bands). The first approach raises the complexity of the conformity assessment procedure, which leads to an increase its cost, time of measurement, there is a need for highly qualified personnel involved. The second approach also has significant disadvantages, since the volume measuring operations increases in Nm times (N is the number of monitoring objects, and m is the number of parallel observations). Therefore, the direct averaging of the results is redundant, and the correctness of the decision depends on the location of the value of the monitoring quantity and the uncertainty associated with the values of the tolerance interval. In this case, there may be a range of possible values of the measured quantity, where the inaccuracy of the measurement does not affect the correctness of the decision on compliance, that is, the actual state and decision on conformity coincide (can be seen in Fig. 3). When implementing the third approach by entering an acceptance interval, the limits of which shifted from the limits of the tolerance interval (shifted to the middle of the tolerance interval) to guard band

$$w = U = 2u$$

reduces the probability of wrong decision making about compliance with the requirements, as shown in Fig. 1. This decision rule is set by default in [7]. However, setting up guard bands between the limits of the tolerance interval and acceptance interval leads to producer losses that, under certain conditions, can be substantial. In doing so, account must be taken of the probability density function (PDF) parameters of possible values of the monitoring quantity.

Thus, a decisive rule of suitability or unfitness object must take into account both the possible uncertainty of the result and the requirements for the cost and complexity of the implementation of the inspection procedure, as well as to the given reliability.

Figure 1. The acceptance interval is formed by decreasing the tolerance interval on each side by the value of the extended uncertainty

So the purpose of work is the development of an adaptive algorithm, the application of which provides a decision to assess the conformity of the monitoring object in accordance with the requirements.

3. Choice of the method of increasing the probability of decision-making

In an inspection, unlike measurements, random effects significantly affect the result, not in the entire range of possible values of the monitoring quantity. Therefore, there is a need to reduce this impact within certain limits around limit values of the tolerance interval with the same time decrease in volume additional measurements. To increase the likelihood of a result, it is necessary to apply such a decision procedure, which takes into account the relationship between the parameters PDF of possible values monitoring quantity, random effects during measurement and the length of the tolerance interval. According to the results of the current measurement, a decision should be taken on the continuation or termination of the inspection with the introduction of additional limits, which provides a reduced probability of erroneous decisions at each additional stage of control for normal or rectangular distributions of the random component of the measurement error.

Consider the case with a two-sided tolerance (Fig. 2), with limits T_L and T_U and width $T = T_U - T_L$, which determines the tolerance T.

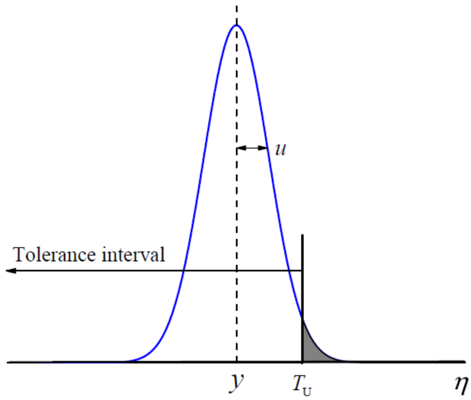

Figure 2. The tolerance interval with the two-sided tolerance interval

The values of the Y parameter with a normal PDF characterizes a certain property of an object after a measurement, which can be represented by the best estimate $\eta_m = y$ and its associated standard uncertainty $u_m = u$. The values Y, which correspond to requirements, lie in the range $\eta \le T_U$. The shaded plane corresponds to the probability of acceptance of false decision about compliance.

The document [5] is introduced measurement capability indicator which characterizes the quality of measurement in relation to the requirements for the monitoring object given by using tolerance interval. This indicator is defined as

$$c_m = \frac{T_U - T_L}{4u_m} = \frac{T}{2U}, \qquad (1)$$

where $U = 2u_m$ – extended uncertainty with a coverage factor of $k = 2$.

Proceeding from the value of the indicator of the measurement capabilities cm, it is possible to calculate the a priori probability of conformity of the monitoring object for the given limits of the tolerance interval (T_L, T_U)

$$p_c = \Phi\left(\frac{T_U - y}{u}\right) - \Phi\left(\frac{T_L - y}{u}\right), \qquad (2)$$

where $\Phi(\cdot) = \frac{1}{\sqrt{2}} \ell^{-(t^2/2)} dt$.

We introduce for a quantity that is in the tolerance interval of the relative value

$$\tilde{y} = \frac{y - T_L}{T}, \qquad (3)$$

that takes values:

$$\tilde{y} = \begin{cases} 0 \text{ for } y = T_L \\ 1 \text{ for } y = T_U \end{cases}.$$

Substituting expression (3) in expression (1) we obtain the dependence, which connects the probability of compliance and the indicator of the measurement capabilities, taking into account the relative value of the tested quantity

$$p_в = \Phi[4c_m(1 - \tilde{y})] - \Phi(-4c_m\tilde{y}) = p_в(\tilde{y}, c_m). \quad (4)$$

It can be shown that values corresponding to the probability $p_c \geq 95\%$ will only be for the relative value of the monitoring quantity in the range of $0{,}45 \leq \tilde{y} \leq 0{,}55$. To expand the range of possible value of the monitoring quantity must increase the value of c_m. The direct way to achieve this is to reduce the uncertainty of measurement и.

However, as practice has shown, not all cases can achieve the above ratios. So, in low-capacity enterprises, for example, confectionery factories, the range of products can vary two or more times a day. In this case, the components of the products may remain unchanged, but the tolerance intervals on them – to change. Since measuring instruments on the processing line remain the same, the instrumental component of measurement uncertainty remains the same also. This leads to a change in c_m, which in turn affects the likelihood of compliance and finally, on the probability of making a correct decision on the results of an inspection.

4. The essence of the adaptive approach

To avoid the above disadvantages and restrictions an adaptive step approach to decision-making on compliance is proposed. At the beginning of the decision-making procedure, based on the actual relationship between the length of the tolerance interval and measurement uncertainty determine the output relative inspection limits $\tilde{y}_{1L}$ and $\tilde{y}_{1U}$, which correspond to the probability of making a decision about the correspondence of $p_c = 95\%$.

On the basis of expression (3) find the relative values of these inspection limits:

$$\tilde{y}_{1L} = \frac{A_{1L} - T_L}{T}; \ \tilde{y}_{1U} = \frac{A_{1U} - T_L}{T},$$

which takes into account the inspection limits for the primary measurement:

$$A_{1L} = \tilde{y}_{1L} + T_L, \qquad (5)$$

$$A_{1U} = \tilde{y}_{1U} + T_L. \qquad (6)$$

With these limit values compare the primary measurement result η_{1m}. If it is within

$$A_{1L} \leq \eta_{1m} \leq A_{1U}, \qquad (7)$$

then with probability $p_c \geq 95\%$, it is decided to match the object (parameter) to the given norms and this procedure of inspection is over.

If inequality (7) is not fulfilled, then pass to the procedure of adaptive definition of inspection limits and comparison with them of the calculated

$$\overline{\eta}_2 = \frac{\eta_{1m} + \eta_{2m}}{2}$$

The values are relative inspection limits find for

$c_{m1} = T/4u_{m1}$,

where $u_{m1} = u_m/\sqrt{2}$.

Absolute values subsequent to one's acceptance limits A_{2L} and A_{2U} are calculated by expressions (5) and (6), where the output values are $\tilde{y}_{2L}$ and $\tilde{y}_{2U}$ which are found for c_{m1}. Again check the implementation of inequality, but with other inspection limits

$$A_{2L} \leq \overline{\eta}_2 \leq A_{2U}.$$

Provided its execution a decision is made on compliance. Otherwise, carry out the third measurement, and the average value of the three measurements $\overline{\eta}_3$ is compared with the newly calculated limits A_{3L} and A_{3U} etc.

The number of additional measurements for each monitoring object depends on the probability of getting $(i + 1)$ average value between the limits of this interval provided that in the previous step, the average value was between limits of the i-th interval, but the probability of compliance was less than 0.95.

The number of objects for which the decision on conformity will be made after the initial measurement is determined by the area under the distribution curve of the possible values of the monitoring quantity between the values of A_{1L} and A_{1U}.

The probability of completing the conformity assessment procedure depends not only on the ratio between $c_m = T/4u_m$ but also on the standard deviation of production technology.

In this way, unlike the existing methods, the conformity of each of the monitoring objects is evaluated.

The number of successive stages is determined by the permissible "residual" probability of making an erroneous decision about the matching of the object. This probability depends on the parameters distribution of controlled size.

5. Adaptive decision-making algorithm

In general, the developed algorithm can be represented as follows.

1. Based on the value of u_m, which depends on the metrological characteristics of the measuring instruments used in the inspection, the initial value

$c_m = T/4u_m$.

2. Establish, on the basis of the dependence (4), the primary relative values of the checked limits $\tilde{y}_{1L}$ and $\tilde{y}_{1U}$.

3. On the basis of expressions (5) and (6), the absolute values of the primary checked limits are found.

4. Compare the measurement result η_{1m} with the absolute values of the primary checked limits in accordance with (7).

If the inequality (7) is satisfied, the object, with a probability of at least 95%, is recognized as corresponding. Otherwise, pass a consistent procedure to reduce measurement uncertainty and to find additional control boundaries.

5. Determine, at the $(i + 1)$ the stage, the mean value as

$$\overline{\eta}_{i+1} = \frac{i\overline{\eta}_i}{i+1} + \frac{x_{i+1}}{i+1}, \left(i = \overline{0,n}\right). \qquad (8)$$

6. Compare the average result with checked limits $A_{(i+1)L}$ and $A_{(i+1)U}$, founded by the formula:

$u_{m(i+1)} = u_m / \sqrt{3+1}$.

If at the n-th stage the probability of a decision on compliance within the T_L and T_U is less than 95%, then a decision is made as to non-compliance object.

Thus, the application of the adaptive algorithm, based on consistent analysis, the main advantage of which is the reduction of the number of additional measurement operations, provides a given reliability of inspection. To implement the algorithm should gradually, depending on the result at the current stage, install additional tolerance intervals. The length of these intervals is determined by the parameters of PDF random variables accompanying measurements of the monitored parameters. The measurement result of the object parameter matched with these limits and a decision is made to continue or ending inspection procedures. If the result of the measurement of the monitored parameter after the current stage of the study did not fall into the zone of additional limits, there is uncertainty and a decision is made about conducting additional studies of this object and comparing the results at each subsequent stage with the "new" values of the additional limits. If the result of the measurement is within the additional limits, an appropriate decision is made according to the adaptive algorithm, and the inspection of this object ends.

Practical use of the algorithm showed that 5 additional stages of research make it possible to reduce the probability of erroneous decisions by at least 3 times, while the increase in measuring operations does not exceed 60%. To achieve the same result by the traditional procedure the number of inspection procedures should be increased by no less than 95%.

6. Conclusion

The presented algorithm allows not to carry out the inspection procedure in full for all possible values of the monitored value, because the incrementally is determined the moment of the primary hit of the monitored quantity inconsistently a measurable interval that can signal on termination further control procedure.

Since uncertainty can be individually estimated for each monitoring object, it is not necessary to change the measuring instruments (or their characteristics). Such an inspection procedure is not weighed down an increase in the cost of inspection, and uncertainty is immediately associated with the likelihood of the object's correspondence. Therefore, with simultaneous increasing the likelihood of making the right decision, in comparison with known methods, does not significantly reduce productivity and slightly increase the cost of production.

7. References

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